$$\mathcal{A} = \left[\frac{1}{2}x \cdot \sin(2x)\right]_0^{\frac{\pi}{4}} - \frac{1}{2}\int_0^{\frac{\pi}{4}} \sin(2x)\,dx$$

$$= \left[\frac{1}{2}x\sin(2x)\right]_0^{\frac{\pi}{4}} - \frac{1}{2}\left[-\frac{1}{2}\cos(2x)\right]_0^{\frac{\pi}{4}}$$

$$= \left[\frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x)\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\cdot\frac{\pi}{4}\cdot\sin\left(\frac{\pi}{2}\right) + \frac{1}{4}\cos\left(\frac{\pi}{2}\right) - \left(0 + \frac{1}{4}\cos(0)\right)$$

$$= \frac{\pi}{8} - \frac{1}{4}$$

$$\mathcal{A} = \frac{\pi - 2}{8}\ cm^2$$